7.SP – Tetrahedral Dice

Alignments to Content Standards: 7.SP.C.8.a 7.SP.C.8.b

Task

Many games use dice which are six-sided and fair (meaning each face on the die is equally likely to land face up). Many games also use the sum of two dice rolled at the same time to determine movement of game pieces. However, not all dice are six-sided. Imagine a game in which two fair four-sided (tetrahedral) dice are rolled simultaneously. These dice are in the shape of a pyramid, and when a die is rolled, the outcome is determined by the side that lands face down. Suppose that for these two dice, the possible values (corresponding to the four sides of the die) that can be obtained from each die are as follows:

Die #1: 1, 2, 3, or 4
Die #2: 2, 4, 6, or 8

a. A certain game determines the movement of players' game pieces based on the SUM of the numbers on the face down sides when two dice are rolled. There are 10 distinct sum values that can occur, and some of those sums occur more often than others.

i. Using an organized list, table, tree diagram, or method of your choosing, develop a list of all 16 possible outcomes (for example, Die #1 = 1 and Die #2 = 2 for a sum of 3; Die #1 = 1 and Die #2 = 4 for a sum of 5; and so on).

ii. From your work in part i, determine the 10 **distinct sum values** that are possible and calculate the probability of obtaining each sum value. Note: as mentioned above, some values will occur more frequently than others.

iii. Using your work in part ii, answer the following questions:
What is the probability of obtaining a sum of 5?
What is the probability of obtaining a sum that is more than 5?

What is the probability of obtaining a sum that is at most 5?
What is the probability of obtaining a sum that is at least 5?
What is the probability of obtaining a sum that is no less than 5?

b. Now consider the case where the DIFFERENCE in the numbers on the face down sides when two dice are rolled is important to the game. Unless the two die values are the same (in which case the difference is 0), the difference for purposes of this game will always be computed as the larger number value rolled minus the smaller number value rolled. In this way, the difference value for any roll of the two dice will always be positive or 0.

i. Using an organized list, table, tree diagram, or method of your choosing, develop a list of all 16 possible outcomes (for example, Die #1 = 1 and Die #2 = 2 for a difference of 1; Die #1 = 1 and Die #2 = 4 for a difference of 3; and so on).

ii. From your work in part i, determine the 8 distinct difference values that are possible and calculate the probability of obtaining each difference value. Note: as mentioned above, some values will occur more frequently than others.

iii. Using your work in part e, answer the following questions:
What is the probability of obtaining a difference of 5?
What is the probability of obtaining a difference that is more than 5?
What is the probability of obtaining a difference that is less than or equal to 5?

IM Commentary

The purpose of this task is to have students develop an organized list, table, etc. to determine all possible outcomes of a chance experiment and then to use this information to calculate various probabilities. Hopefully, students will note that techniques applicable to six-sided dice and sums are also applicable to four-sided dice and differences. With certain approaches, patterns in the listing of possible outcomes will be more easily recognized.

Students will need to make distinctions in the language of "more than" vs. "at least" vs. “at most”, etc. Students should also note that different words can be used to describe the same compound events.

If students are familiar with absolute value, you can also use this terminology in describing the variable in question 2 of this task.

Edit this solution

Solution

a. i.

Sum		Die #2			
		2	**4**	**6**	**8**
Die #1	**1**	3	5	7	9
	2	4	6	8	10
	3	5	7	9	11
	4	6	8	10	12

ii.

X	**Probability**
3	$\frac{1}{16}$
4	$\frac{1}{16}$
5	$\frac{2}{16}$
6	$\frac{2}{16}$
7	$\frac{2}{16}$
8	$\frac{2}{16}$
9	$\frac{2}{16}$
10	$\frac{2}{16}$
11	$\frac{1}{16}$
12	$\frac{1}{16}$

iii. **What is the probability of obtaining a sum of 5?**

$\frac{2}{16} = \frac{1}{8}$ (see distribution)

What is the probability of obtaining a sum that is more than 5?

This includes all outcomes $X = 6$ to $X = 12$. This is $\frac{12}{16} = \frac{3}{4}$

What is the probability of obtaining a sum that is at most 5?

This includes all outcomes $X = 3$ to $X = 5$. This is $\frac{4}{16} = \frac{1}{4}$

Note: this answer is also the complement to the previous question.

What is the probability of obtaining a sum that is at least 5?

This includes all outcomes $X = 5$ to $X = 12$. This is $\frac{14}{16} = \frac{7}{8}$

What is the probability of obtaining a sum that is no less than 5?

This includes all outcomes $X = 5$ to $X = 12$. This is $\frac{14}{16} = \frac{7}{8}$

Note: this is another way of saying "at least 5" as in the previous question, hence the similar answer.

b. i.

Difference		Die #2			
		2	**4**	**6**	**8**
Die #1	**1**	1	3	5	7
	2	0	2	4	6
	3	1	1	3	5
	4	2	0	2	4

ii.

X	**Probability**

0	$\frac{2}{16}$
1	$\frac{3}{16}$
2	$\frac{3}{16}$
3	$\frac{2}{16}$
4	$\frac{2}{16}$
5	$\frac{2}{16}$
6	$\frac{1}{16}$
7	$\frac{1}{16}$

iii. **What is the probability of obtaining a sum of 5?**

$\frac{2}{16} = \frac{1}{8}$ (see distribution)

What is the probability of obtaining a sum that is more than 5?

This includes all outcomes $X = 6$ to $X = 7$. This is $\frac{2}{16}$

What is the probability of obtaining a sum that is less than or equal to 5?

This includes all outcomes $X = 0$ to $X = 5$. This is $\frac{14}{16}$

Note: this answer is also the complement to the previous question.