A-REI Canoe Trip

Alignments to Content Standards: A-REI.A.2

Task

Jamie and Ralph take a canoe trip up a river for 1 mile and then return. The current in the river is 1 mile per hour. The total trip time is 2 hours and 24 minutes. Assuming that they are paddling at a constant rate throughout the trip, find the speed that Jamie and Ralph are paddling.

IM Commentary

The goal of this task is to set up and solve an equation involving a simple rational expression. This gives rise to a quadratic equation which has only one meaningful solution in terms of the context. It is important that students understand that the speed of the water current is subtracted from Jamie and Ralph's 'still water' speed when they are going against the current and is added to when they are going with the current. This is a reasonable assumption which is certainly a good approximation but if students have not worked with this type of rate problem before they may require guidance modeling the speed of the canoe with and against the current.

In the solution, the quadratic function which is eventually solved to find the Jamie and Ralph's speed has one valid solution and another solution which is negative and so does not make sense in the context. It is important to note that within this context, a negative speed is not the only type of speed that makes no sense: any speed less than 1 mile per hour does not make sense either as Jamie and Ralph would never make it upstream paddling at this rate.

Edit this solution

Solution

Suppose we let x denote the speed, in miles per hour, that the canoe would travel with no current. When they are traveling against the current, Jamie and Ralph's speed will be $x-1$ miles per hour and when they are traveling with the current their speed will be $x+1$ miles per hour. The trip upstream will take $\frac{1}{x-1}$ hours and the trip downstream will take $\frac{1}{x+1}$ hours. There are $\frac{2}{5}$ of an hour in 24 minutes so the total trip lasts for $2\frac{2}{5}$ hours giving us

$$\frac{1}{x-1}+\frac{1}{x+1}=\frac{12}{5}.$$

Multiplying both sides of the equation by $(x-1)(x+1)=x^2-1$ gives

$$(x+1)+(x-1)=\frac{12}{5}(x^2-1).$$

This equation simplifies to $\frac{12}{5}x^2-2x-\frac{12}{5}=0$ or, after further manipulation,

$$6x^2-5x-6=0.$$

We can use the quadratic formula to solve for x:

$$x=\frac{5\pm\sqrt{25+144}}{12}.$$

We have $\sqrt{169}=13$ so the two solutions are $x=\frac{5\pm13}{12}$ or $x=\frac{3}{2}$ and $x=-\frac{2}{3}$. The second solution does not make any sense in this context as the speed cannot be negative. So Jamie and Ralph are paddling at a rate of $\frac{3}{2}$ miles per hour. Going upstream, the trip takes longer against the current and going with the current the trip is shorter.